

On statistical distribution of peak currents of lightning discharges

О статистическом распределении пикового тока молниевых разрядов

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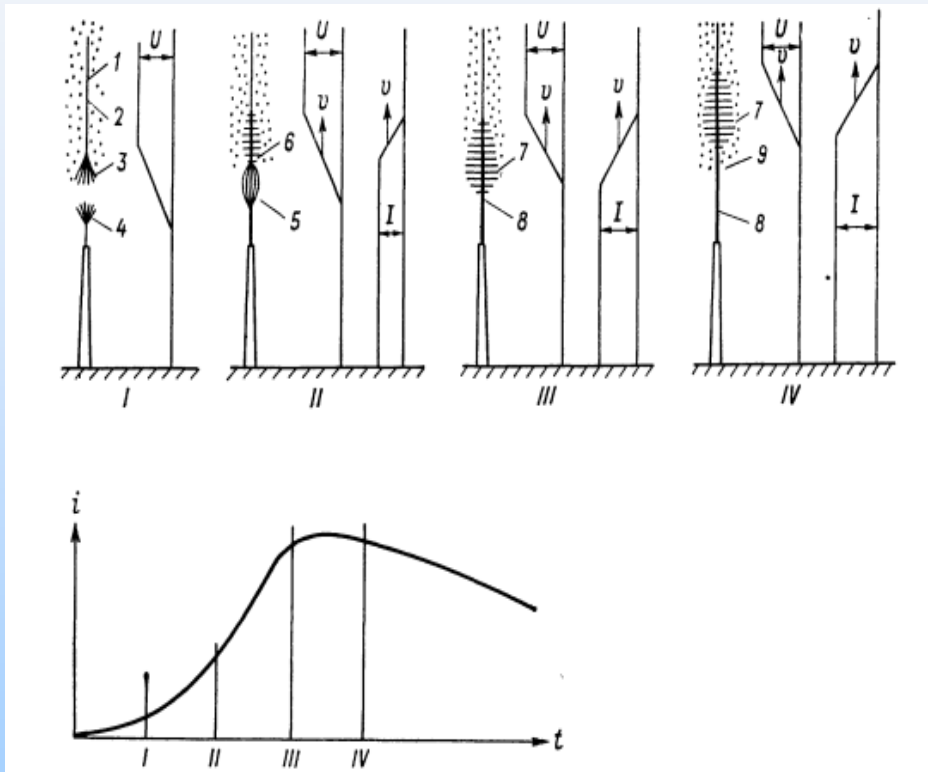
evgeny.mareev@gmail.com

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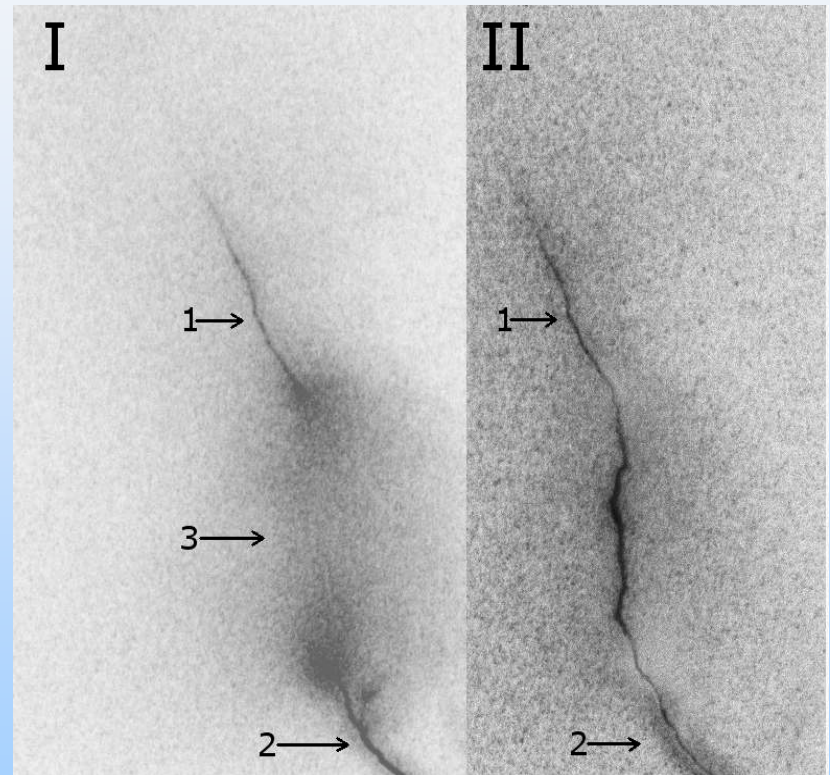
- Characterization of lightning currents
- Statistics of lightning peak currents (PC)
- Lognormal PC distribution: experiments
- Lognormal PC distribution: theory
- The median (50%) and severe (1%) lightning currents at ground
- On the upper limit of peak current of first return strokes in negative lightning flashes
- Unresolved problems and perspectives

Downward and upward leaders; lightning attachment

Bazelyan et al. (1978)

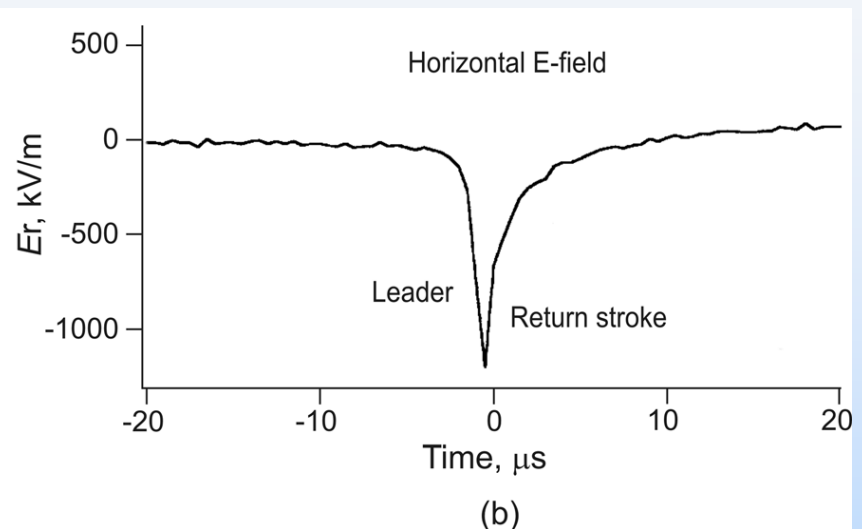
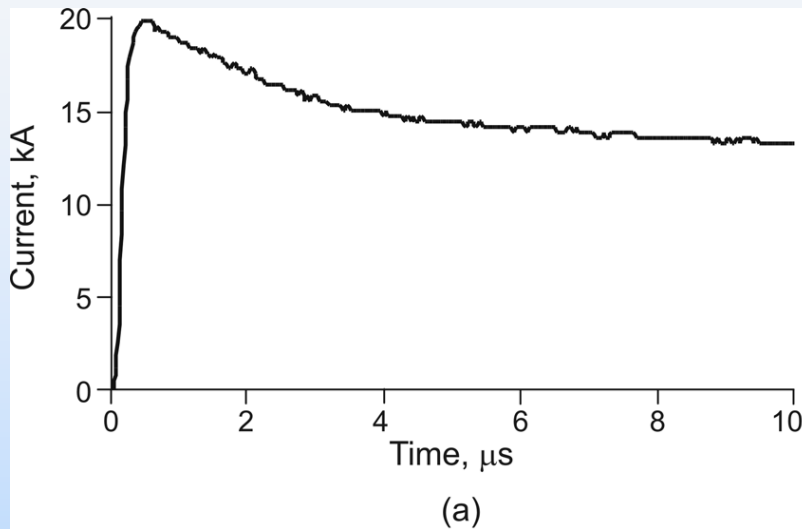


Kostinsky et al., JGR (2016)



Two 4Picos frames showing the break-through phase (I) and later return-stroke stage (II) of a negative discharge to ground generated by the cloud of artificially-charged water droplets. The exposure time for each frame is 100 ns and the time interval between frames is 2 μ s. Labeled are the electrodeless downward negative leader 1, upward positive leader 2, and the common streamer zone 3. Image (II) was contrast-enhanced more than image (I), to improve its visualization.

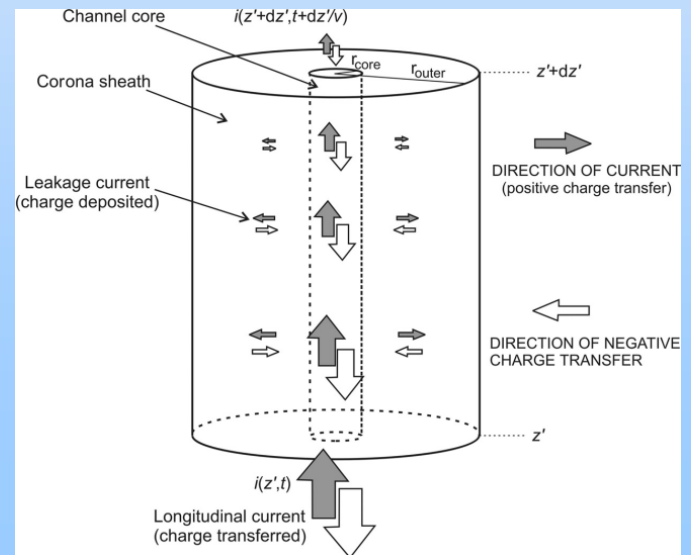
Return stroke current



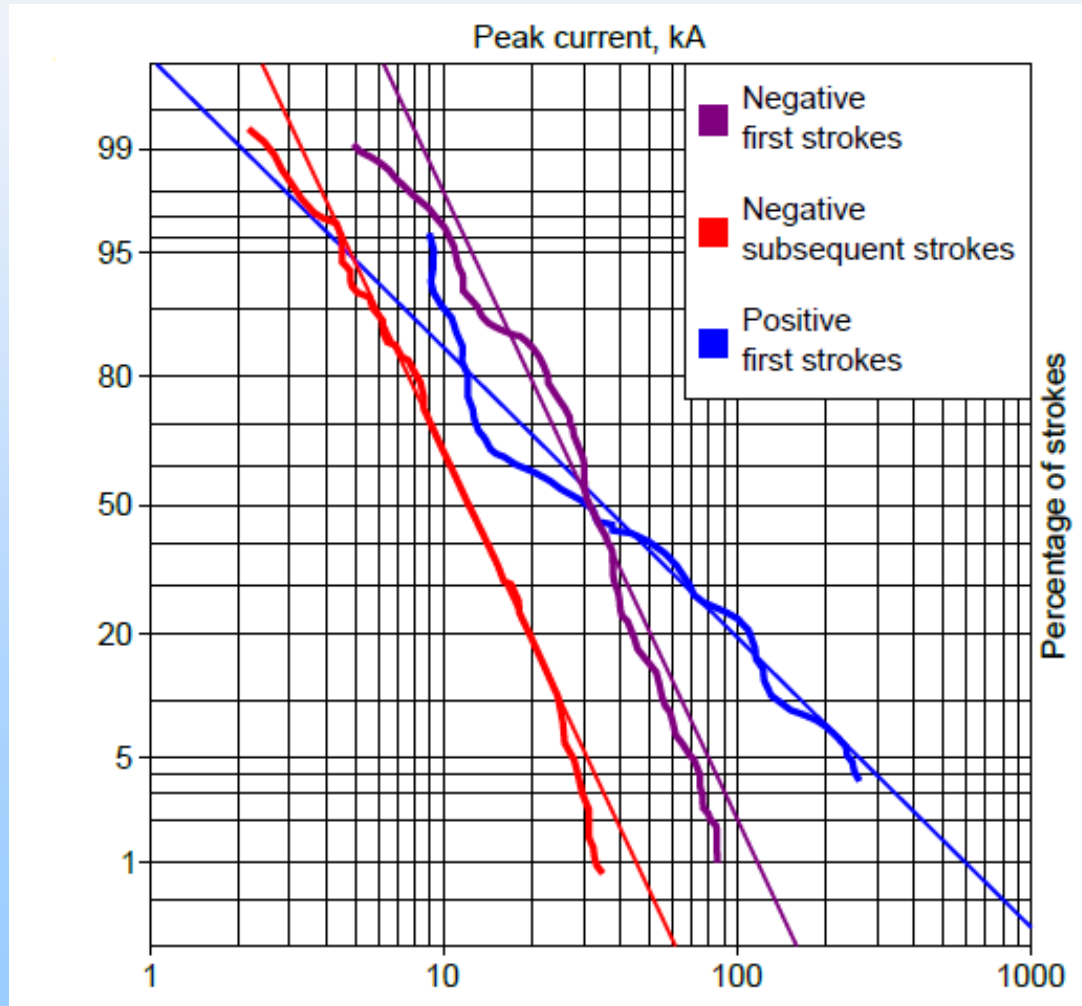
Miki et al. (2002)

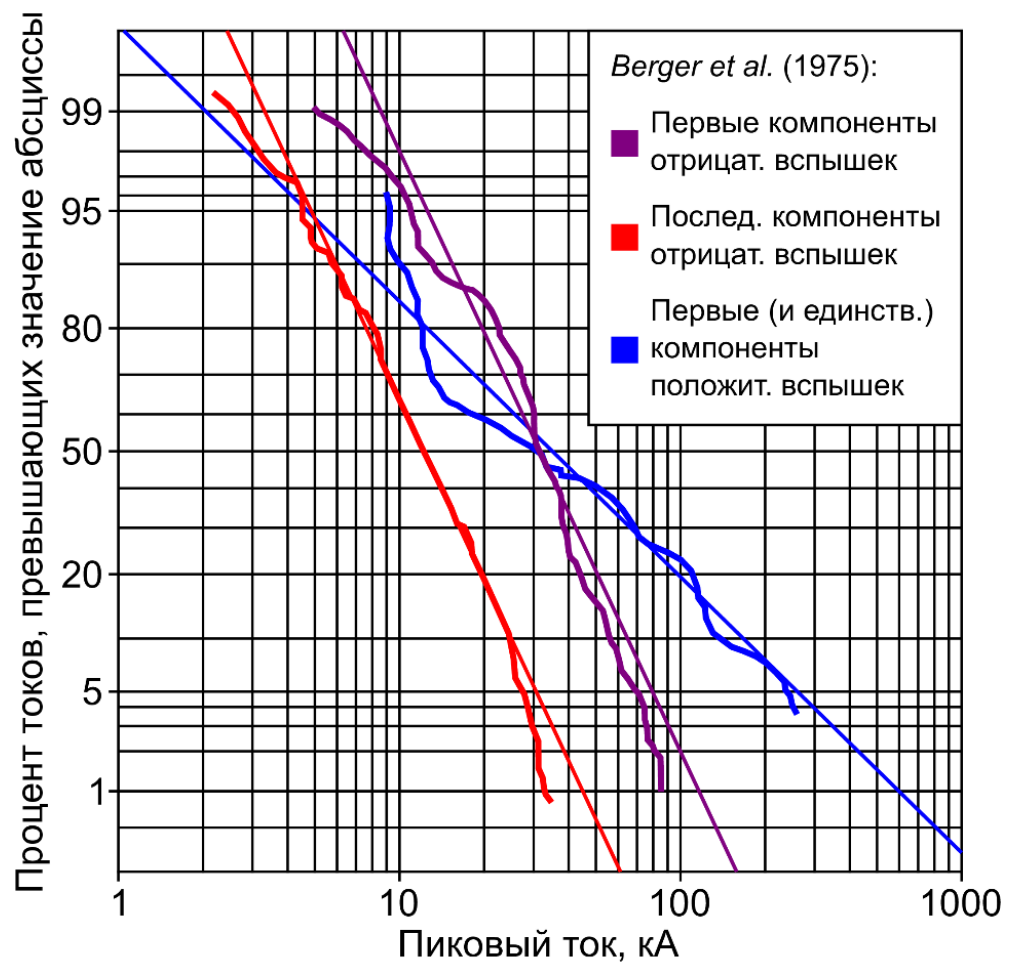
Return stroke current at the channel base and (b) corresponding horizontal (radial) electric field 0.1 m from the triggered lightning channel core

Maslowsky and Rakov (2006)



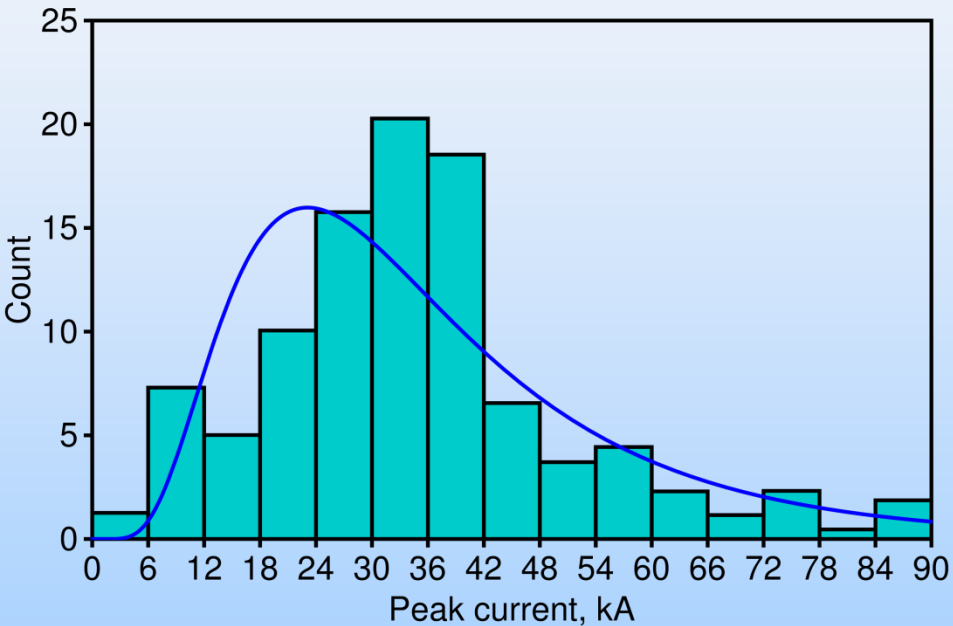
Berger et al. (1975) data for peak current



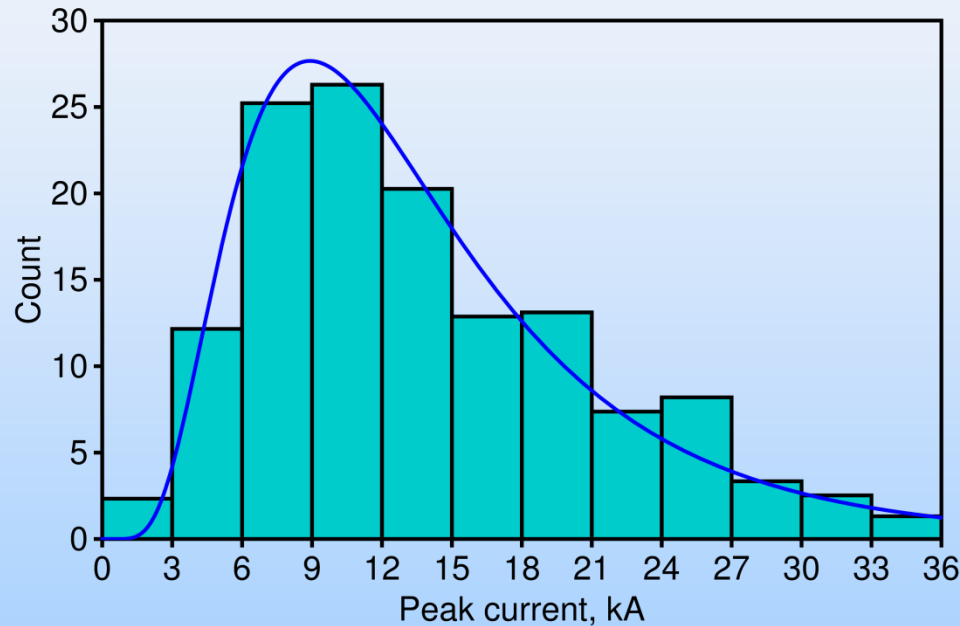


Histograms derived from Berger et al. (1975)

Negative first strokes



Negative subsequent strokes



Lightning PC – Berger's Distributions

Lightning peak currents for **first strokes** vary by a factor of 50 or more, from about 5 to 250 kA.

The probability of occurrence of a given value **rapidly increases up to 25 kA** or so and then slowly decreases.

Statistical distributions of this type are often assumed to be **lognormal**.

$$P(I) = \frac{1}{\sqrt{2\pi\sigma I}} \exp \left\{ -\frac{(\ln I - \ln I_0)^2}{2\sigma^2} \right\}$$

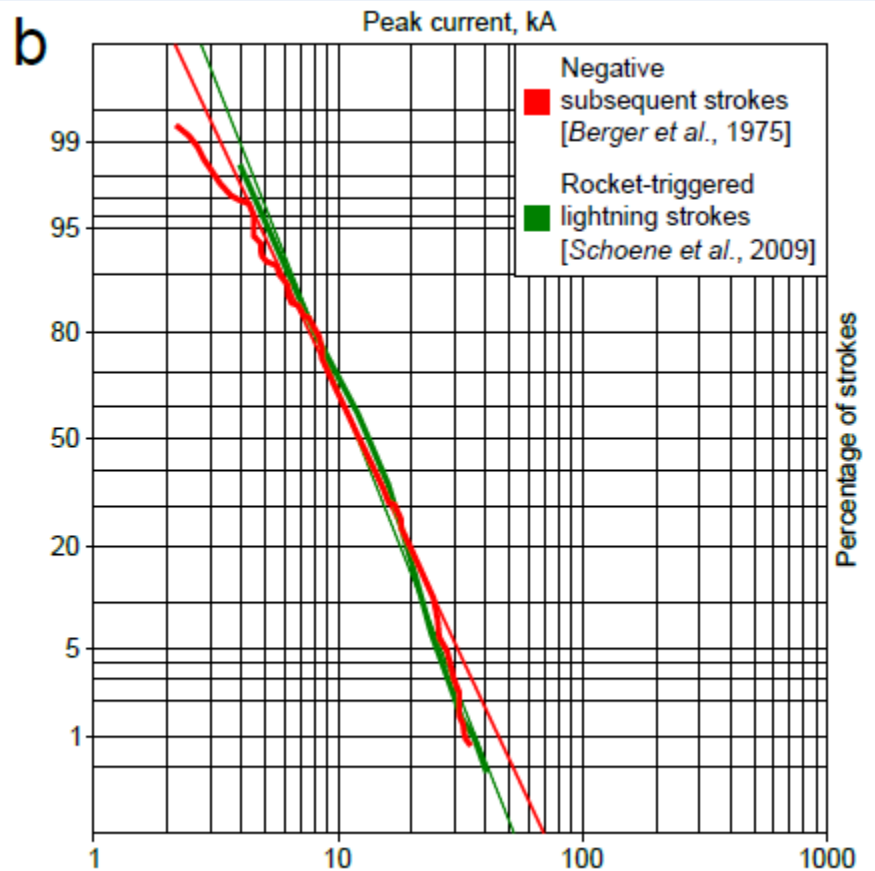
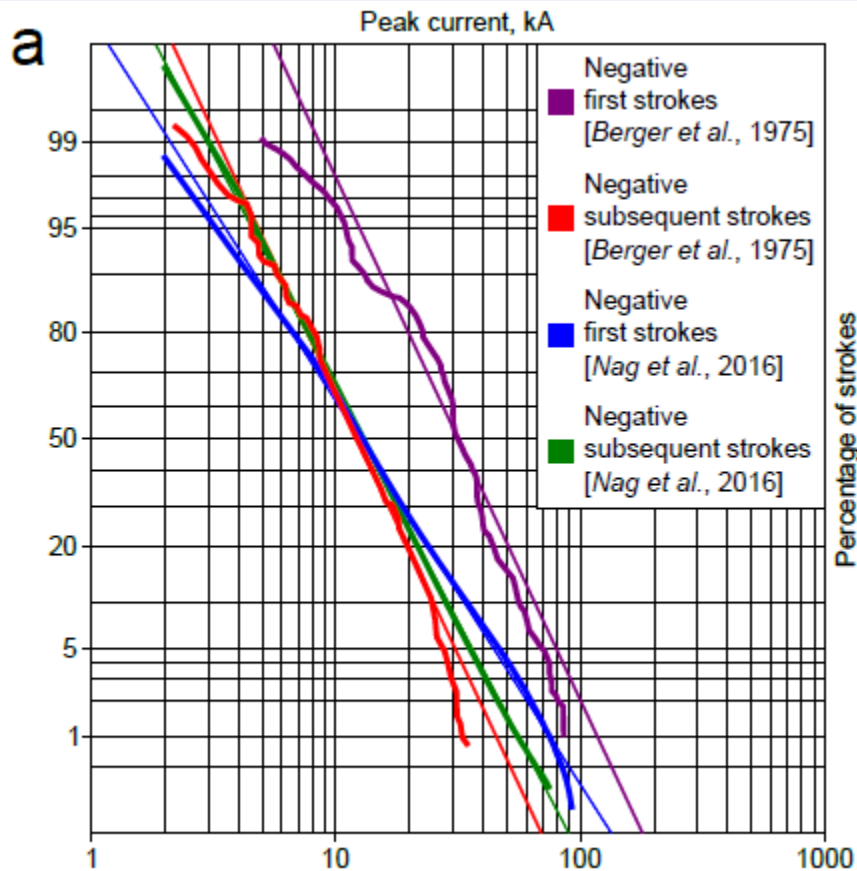
Cumulative statistical distributions of lightning peak currents, giving percent of cases exceeding abscissa value, from direct measurements in Switzerland (Berger et al. 1975). The distributions are assumed to be lognormal and given for (1) **negative first strokes** (N=101), (2) **positive first strokes** (N=26), and (3) **negative subsequent strokes** (N=135).

Return-Stroke Parameters Derived from Current Measurements

*Direct current measurements by K. Berger and co-workers in **Switzerland** remain the primary reference for both lightning research and lightning protection studies. Berger's peak current distributions are generally confirmed by recent direct current measurements, particularly by those with larger sample sizes obtained in **Japan (N=120)**, **Austria (N=615)**, and **Florida (N=165)**.*

*From direct current measurements, the median return-stroke peak current is about **30 kA** for negative first strokes and typically **10-15 kA** for subsequent strokes. Additional measurements are needed to determine more reliably the tails of the statistical distributions.*

Сравнение с данными NLDN и тригг. МОЛНИИ



Lightning current characteristics

	Measured Values		Modeled Values	
	50%	1%	50%	1%
RETURN STROKE PARAMETERS				
NEGATIVE FIRST STROKES				
(a) Peak current (kA)	30	150	32	160
(b) Time from zero to current peak (μs)	5	30	6	6
(c) Maximum rate of current rise (kA/ μs)	100	400	100	500
(d) Time to decay from peak to half-peak value (μs)	70-80	300	75	80
(e) Charge Transfer (C)	5	40	5.5	27
POSITIVE FIRST STROKES				
(a) Peak current (kA)	35	500	35	350
(b) Time to current peak (μs)	10-20	150	11	11
(c) Maximum rate of current rise (kA/ μs)	100	400	100	500
(d) Time to decay to half-peak value (μs)	†	†	30	40
NEGATIVE SUBSEQUENT STROKES				
(a) Peak current (kA)	10-15	50	11	56
(b) Time to current peak (10-90 percent) (μs)	0.3-0.6	9	0.6	0.6

MEASURED LIGHTNING CURRENT CHARACTERISTICS RECOMMENDED BY THE PRESENT STUDY AND MODELED VALUES

Gamerota et al., IEEE Trans. Electrom. Compat., 2012

Summary of Lightning peak current values

The median (50%) and severe (1%) lightning currents at ground appear to be:

30 kA and 150 kA for negative first strokes,
10-15 kA and 50 kA for negative subsequent strokes,

35 kA and 500 kA for positive first strokes (as follows from lognormal distributions)

Основные соотношения теории

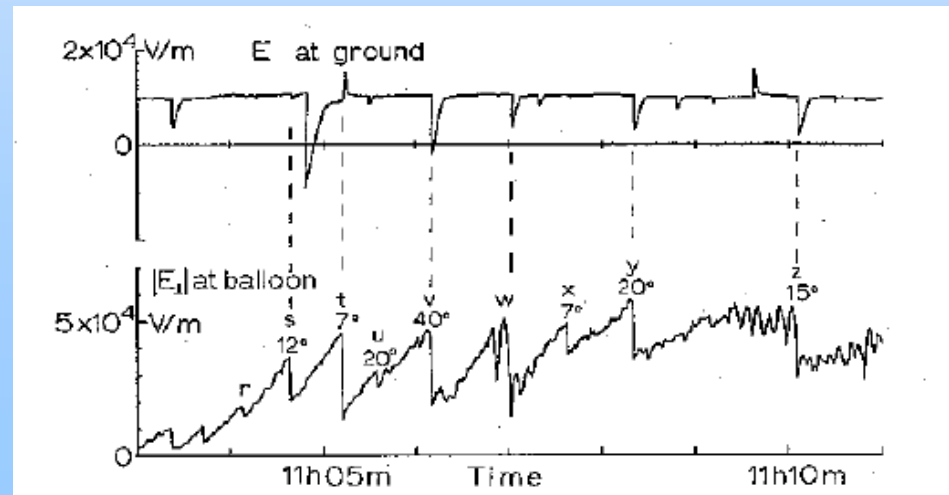
$$1 - P(E, \tau_1 + \tau_2) = (1 - P(E, \tau_1))(1 - P(E, \tau_2))$$

$$P(E, \tau) = 1 - (1 - P(E, \tau'))^{\tau/\tau'}$$

$$F_E(\varepsilon) = 1 - \lim_{\eta \rightarrow \varepsilon+0} \exp\left(\frac{1}{\tau} \int_{E(0)}^{\eta} \ln(1 - P(E', \tau)) \frac{dt}{dE}(E') dE'\right)$$

$$E(t) = E(0) + kt$$

$$P(E, \tau_0) = \begin{cases} 0, & 0 \leq E \leq E_0, \\ \frac{E - E_0}{E_1 - E_0}, & E_0 < E \leq E_1, \\ 1, & E > E_1; \end{cases}$$



Peak current distribution and its genesis

$$P(I) = \frac{1}{\sqrt{2\pi}\sigma I} \exp \left\{ -\frac{(\ln I - \ln I_0)^2}{2\sigma^2} \right\}$$

$$F_E(\varepsilon) = \begin{cases} 0, & \varepsilon \leq E_0. \\ 1 - \exp \left(-\frac{1}{k\tau_0} \left(\varepsilon - E_0 + (E_1 - \varepsilon) \ln \frac{E_1 - \varepsilon}{E_1 - E_0} \right) \right), & E_0 < \varepsilon \leq E_1. \\ 1, & \varepsilon \geq E_1. \end{cases}$$

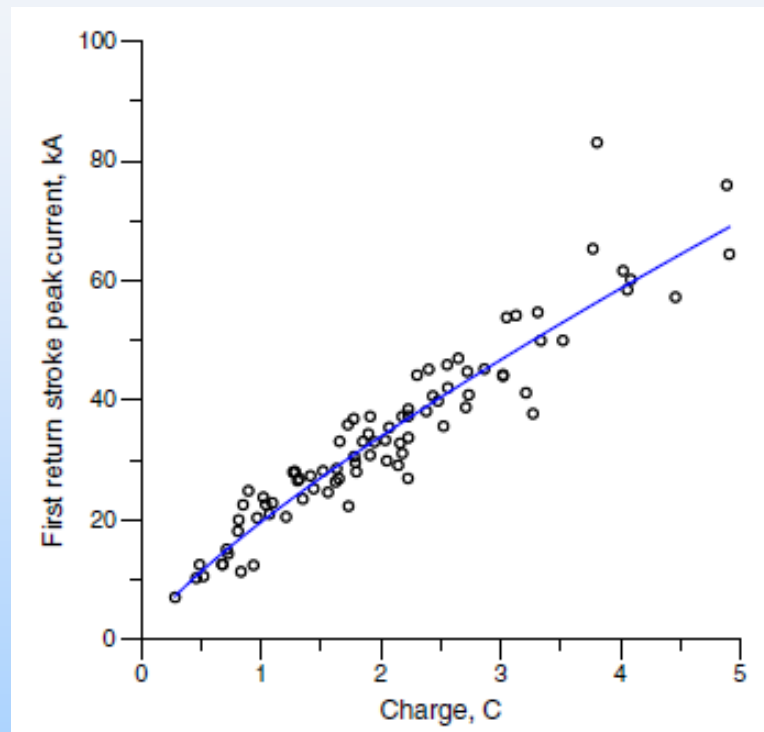
On the upper limit of peak current of first return strokes in negative lightning flashes

$$I_{peak} = B E$$

$$I_{peak} = 2.44 E^{0.967}$$

$$I_{peak} = 19.6 Q^{0.78}$$

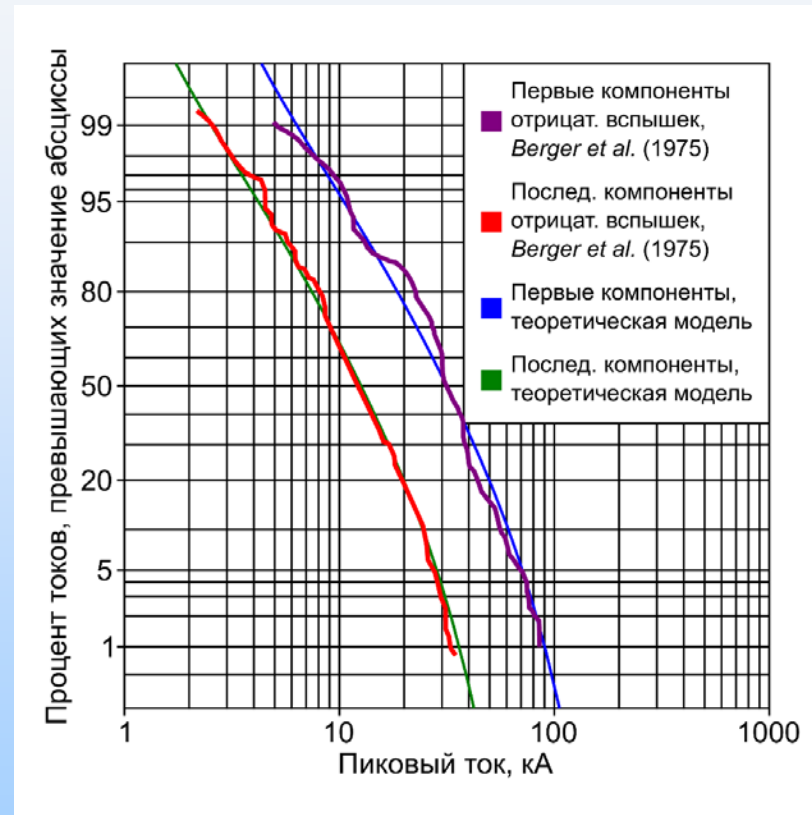
$$I_{peak} = 10^{-6} V^{0.967}$$



Maximum peak current is about 300 kA in temperate regions and about 450 kA–500 kA in the tropics

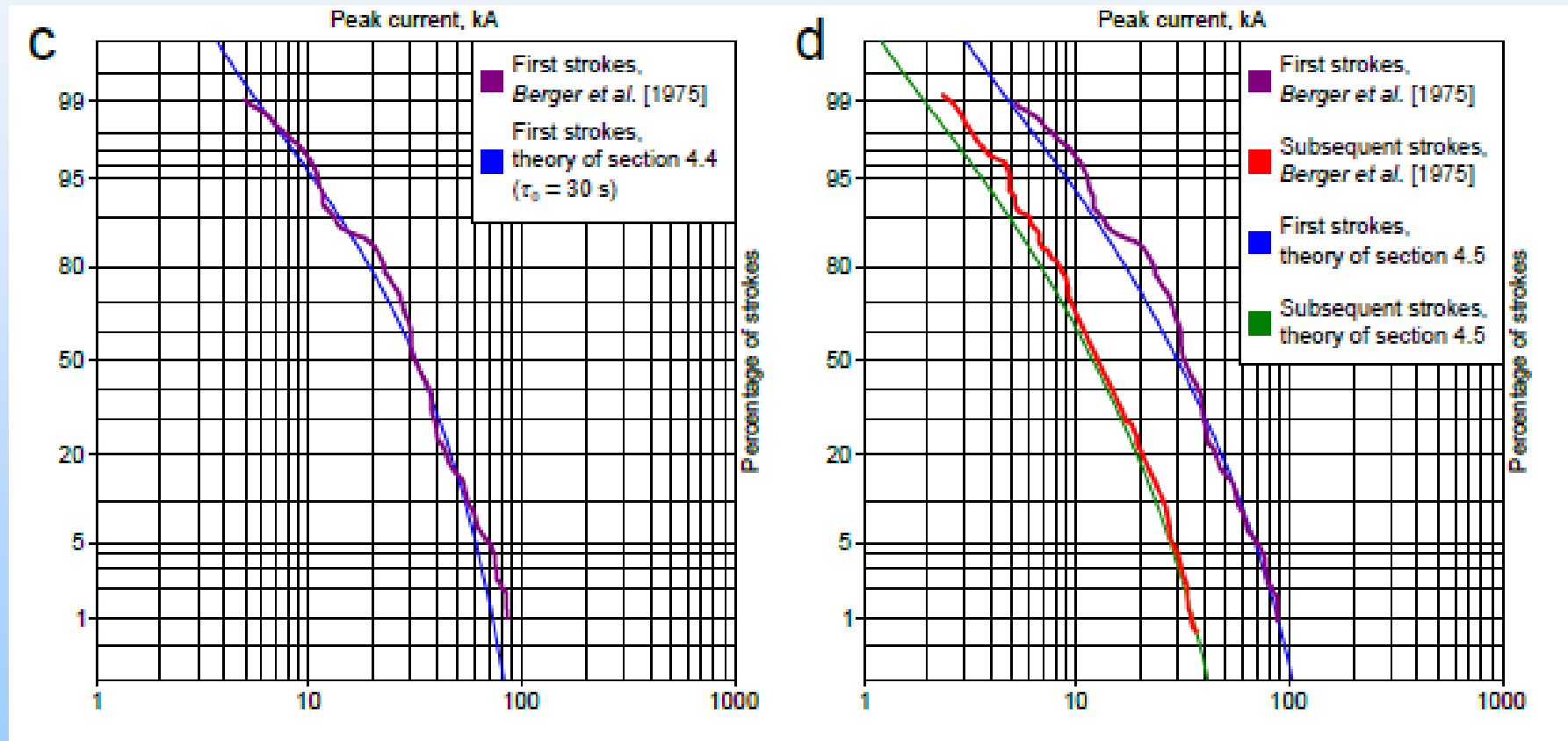
Cooray and Rakov, 2012

Сравнение теории с экспериментальными данными



Сравнение распределения пиковых токов первых и последующих компонент отрицательных всплесков из работы Бергера и модельных распределений

Statistical distributions of lightning peak currents: Why do they appear to be lognormal?



N. Slyunyaev, E. Mareev, V. Rakov, and G. Golitsin, 2017

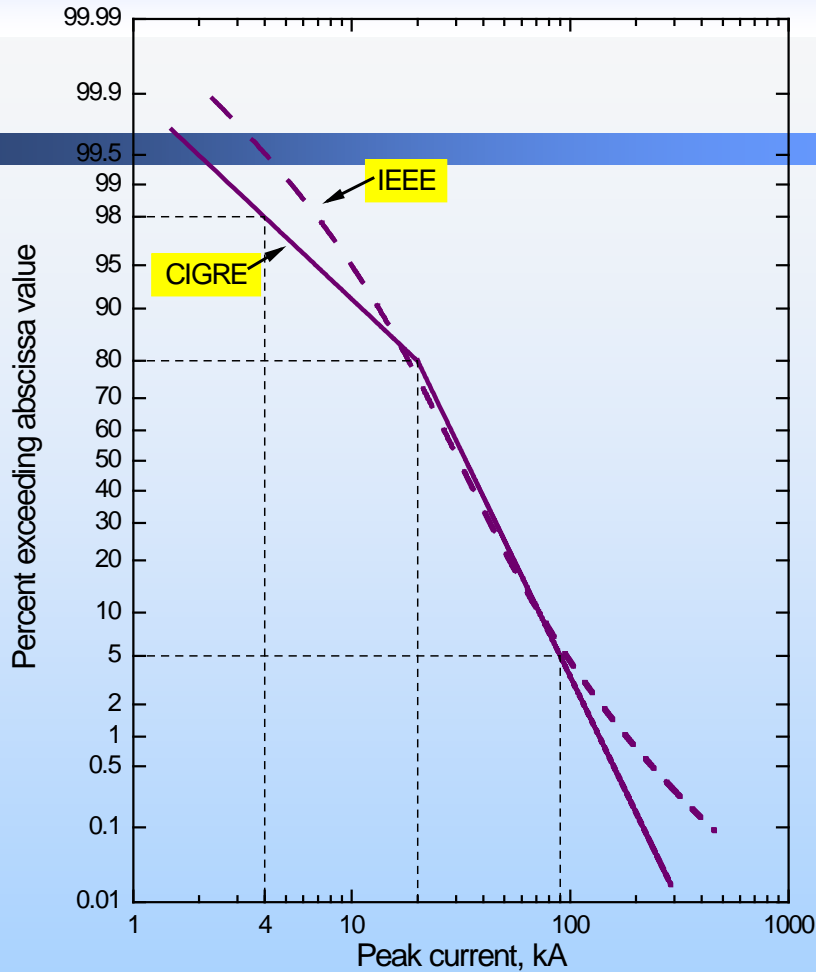
Conclusions

- The CIGRE Brochure #549 (2013) recommends the use of direct current measurements.
- The median (50%) and severe (1%) lightning currents at ground appear to be:
 - 30 kA and 150 kA for negative first strokes,
 - 10-15 kA and 50 kA for negative subsequent strokes,
 - 35 kA and 500 kA for positive first strokes (as follows from lognormal distributions)
- The maximum directly measured peak current to date is about 300 kA, and it is for positive lightning.
- For negative lightning, the maximum directly measured peak current is about 200 kA.

Conclusions

- For 150 kV/m as the largest background electric field the maximum peak current is about 300 kA in temperate regions and about 450 kA–500 kA in the tropics
- The distributions of peak currents directly measured at instrumented towers are compared to those of peak currents reported by the U.S. National Lightning Detection Network (NLDN) and peak currents measured in rocket-triggered lightning experiments; all those appear to be nearly lognormal.
- To explain the observed peak current distributions, a simple model based on the large-scale electric field evolution prior to lightning flashes is developed. With the help of only the most general assumptions concerning the probability of occurrence of a lightning discharge, it is shown that the distribution of the pre-discharge large-scale electric field is close to lognormal in a certain range. The semi-empirical relationships between the peak current, leader potential, and large-scale electric field for negative first strokes then yield that the peak current obeys a distribution of the same type.

Lightning PC – IEEE and CIGRE Distributions



For the **CIGRE distribution**, 98% of peak currents exceed 4 kA, 80% exceed 20 kA, and 5% exceed 90 kA.

For the **IEEE distribution**, the “probability to exceed” values are given by the following equation

$$P_I = \frac{1}{1 + \left(\frac{I}{31}\right)^{2.6}}$$

where P_I is in per unit, and I is in kA. This equation applies to values of I up to 200 kA. The median (50%) peak current value is equal to 31 kA.

Cumulative statistical distributions of peak currents for negative first strokes adopted by IEEE and CIGRE (N = 408). Taken from CIGRE Report 63 (1991).

Peak current, I, kA (IEEE distribution)	5	10	20	40	60	80	100	200
Percentage exceeding tabulated value, $P_I \cdot 100\%$	99	95	76	34	15	7.8	4.5	0.8